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STOKES' DRIFT AND THE SACRED-COW SYNDROME

by

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INTRODUCTION

This paper is intended to draw the attention of those physical oceanographers from ICES member-countries who specialise in current measurements to the dilemma that we are in as far as the interpretation of our records in terms of residual drift is concerned. Put briefly, one side of the dilemma is that some theoreticians claim that estimates of residual drift calculated from a fixed point in space (the Eulerian frame of reference) can be misleading as far as the drift from point-to-point in space (the Lagrangian frame of reference) is concerned. This is because the Stokes' drift factor, which must be added vectorially to the Eulerian drift estimate before a satisfactory approximation to the Lagrangian drift is obtained, can be as large, if not larger, than the Eulerian drift estimate itself.

Ever since this point of view was aired by Professor Longuet-Higgins at the ICES Dublin meeting of 1969, (Longuet-Higgins 1972) European physical oceanographers have been aware that it is likely to be particularly relevant as far as measurements made in various parts of the seas around Britain are concerned. Consequently it has been:

i. taken into account as an aside in the report of the ICES Working Group on Pilot Current Meter Stations (Ramster (Ed) In Prep);

ii. developed in considerable detail for the Irish Sea region as part of a mathematical modelling project;

iii. worked-up in detail in terms of the practical aspects of the computer programs that were needed for calculating the velocity gradients at a given point in time for a given area, and

iv. investigated on 3 or 4 occasions in the field in that attempts have been made to compare the calculated and measured values of the Stokes' drift factor for given areas.

Now, however, it has been suggested to us informally, but very forcibly, that the residual current specialists have made Stokes' Drift "a sacred cow" and given it more importance in their programmes of work than it ought to have from a purely scientific point of view. Which of these viewpoints is correct is the dilemma that we need to resolve as quickly as possible.

We accept without qualification that in our case the general charge is true: we have indeed attached very great importance to Stokes' Drift at the Lowestoft Laboratory because residual drift patterns are one of the corner stones of our fisheries oceanography programme. However, we are very intrigued by the idea that we have raised this topic to the status of a "sacred cow" and see the need to review the general situation so that we can find out for ourselves how far we have progressed since 1969 and also allow some form of objective "Peer Review" to occur.

A REVIEW OF THE PUBLISHED WORK TO DATE

It is not intended in this paper to provide a very detailed review of the Stokes' Drift work that has been published since 1969 but rather to list the relevant papers and highlight their main points in order to be absolutely clear about the various ways in which Stokes' Drift has been interpreted. We have listed in Annex 1 the various formulae used in each case.

In the same volume in which Longuet-Higgins stated the case for carefully evaluating the Stokes' Drift factor relating to any estimates of residual drift calculated from moored current meter data, Hill and Ramster (1972) followed his suggestions and applied them to data collected in the western Irish Sea. They found that at one station the addition of the Stokes' Drift factor increased the magnitude of the Eulerian drift by 58% but that it did not significantly alter its direction. At five other stations the Stokes' drift was deemed insignificant. With the benefit of hindsight it is now clear that the calculations adopted in this instance were not rigorous enough so that only "order of magnitude" estimates were obtained.

Hunter (1972) estimated the Stokes' drift from the results of his mathematical model of the Irish Sea by defining the Eulerian residual to be the mean velocity and the Lagrangian drift to be the mean transport divided by the mean depth. Hence he was able to produce charts of Eulerian and Lagrangian vectors generated by a tidal input. The Stokes' drift, being the difference between the Lagrangian and Eulerian drift, was also derived for each point of his model grid. Figure 1 shows the Stokes' factor chart produced by Hunter's model and from this it can be seen that in the southern and eastern parts of the region magnitudes of 0.5-3 cms^{-1} occur while in the north and west insignificant magnitudes are found. Unfortunately, it is not generally accepted that Hunter's formulation of the Lagrangian drift is correct and this has restricted the general application of his work.

A rather different approach to the topic was taken by Dooley (1974a) who compared drogue and current meter measurements in relatively shallow waters in the northern North Sea. He listed initially the drawbacks of parachute drogue exercises and was careful to use only drogue measurements taken during calm conditions. He found nonetheless that the patterns of movement were quite different and that estimates of the Stokes' velocity calculated solely from the characteristics of the tidal wave did not bring them into agreement. In summing up his results he states "for an area off the east coast of Scotland an ill-defined relationship exists between Lagrangian and Eulerian movements. The component of flow which is related to a travelling wave is small compared with the rectified flow generated by variable bottom topography. Care must be taken in interpreting small residual flows as measured by current meters since local effects giving rise to asymmetry of the tidal stream may be dominant." This view was written into the report of the ICES Working Group on Pilot North Sea Current Meter Networks.

At about the same time yet another approach to the general problem was being taken by Talbot et al (In prep) in their interpretation of current meter data gathered off the coast of Yorkshire. They compared Eulerian estimates of residual drift from 3 stations situated at various distances from the coast with estimates of mass transport derived from the velocities recorded at each station and coastal tide gauge records. In other words they took account of the Stokes' drift factor by estimating the actual amount of water flowing through each of their moored current meter positions during each tidal cycle.

In most of the 17 comparisons the agreement as far as residual direction was concerned was excellent and there was a small systematic difference between the two sets of residual velocities. This suggests therefore that in this particular area at least the Stokes' factor is relatively small and that it could be calculated from the known characteristics of the tidal wave. The authors do make the point however that they have not been able to use data from all parts of the water column but have had to assume that the two velocity values they have at each station are representative of a constant fraction of the tidal water column.

In 1973 the present authors reported to ICES the results of a "Stokes triangle" exercise (as suggested by Longuet-Higgins in 1969) that had been coupled with the tracking within the triangle of a parachute drogue system. This was, to our knowledge, the first time such an exercise had been done and the primary reason for our writing at that time was to spread the news of the practical difficulties involved and the type of results that might be expected. We also reported that we had evolved a computer routine for calculating the velocity gradients that took account of all the data collected at the stations of the triangle at a given time. Unfortunately, as Dooley had suggested earlier, windage on the surface buoy was found to be very important and it was felt that a shallow water "Swallow float" (Swallow, 1955) was needed to overcome this difficulty. Furthermore, the calculations suggested that the Stokes' factor at the time of the exercise, which was carried out incidentally in the same region as Dooley's earlier work, was of the same order as the Eulerian residuals measured.

It was noted that there seemed to be no links between the character of the calculated Stokes' drift and the progressive tidal wave in the area.

The 1973 paper was criticised by Dooley (1974b) principally on the grounds that complex bottom topography will lead to spatial variability within the region of such a triangle of moorings and this will effectively dominate the field measurements. He also pointed out other possible weaknesses in the approach that had been taken and went on to suggest that "reliable estimation of Stokes' drift is dependent primarily on the homogeneous distribution of tidal current. This demands that current meter records must first be examined for the quality of their harmonic constituents and once the above condition has been verified Stokes' drift can be reliably estimated." He then proceeded to examine 2 sets of data and compute Stokes' drift from the values of the M_2, S_2 and O_1 constituents in the data. Later he extended his system to the whole of the region lying east of the Scottish coast to as far as 1^{O}E by computing the drift from the known character of the tidal wave in the region and made the telling point that Spring-Neap fluctuations need to be taken into account in any attempt at future calculation of Stokes' drift estimates.

It is very clear that the second of Dooley's papers has much more in common with the approach taken by Hunter and Longuet-Higgins than his first one. He now appears to suggest that the importance of bottom topography is rather less in general terms than he had first supposed and that reliable estimates of the Stokes' factor can be computed for large sea areas on a routine basis.

For our part we accept as justified the criticisms made of various aspects of our 1973 paper and reiterate that we were intent primarily on publicising the type of field exercise that we felt to be necessary at that time. Since then we have carried out two more Stokes' triangle exercises in the central North Sea and western Channel respectively; the first of these taking place in almost totally calm conditions. We have also modified the formula which we used in our previous paper in order to allow an hourly comparison of calculated and measured drift instead of the tidal mean comparison previously employed. Furthermore as a result of considering the great spatial variability found in the first field experiment we have also developed a more accurate "second order formula" for calculating the hourly values of Lagrangian drift. The details of these formulae are given in Annex 2.

We now find, however, that in both recent field experiments there is, if anything, slightly better agreement between the drift of the parachute drogue and calculated Eulerian residuals than exists between the drogue drift and the calculated Lagrangian estimates (Figures 2 and 3)!! In neither of these more recent cases did we have the marked spatial variability within the triangle of moorings that was so apparent off the east coast of Scotland and consequently there was little difference in fact between the Lagrangian drift calculated by the second order formula and that calculated from the Longuet-Higgins formula. This lack of spatial variability must, we feel, offset to some extent Dooley's strictures on our original experimental set-up. We agree with him, however, that the parachute drogue *per se* is the weakest part of the experimental design and reiterate our plea for the design of a shelf-sea Swallow float that would move along, within acceptable limits at least, at or close to the height above the seabed of the moored current meters.

Finally, in this review we would like to draw attention to a residual drift model of the southern North Sea by one of us (Durance, 1975) which suggests that the Stokes' velocity, as calculated in the sense previously defined by Hunter, could play an important role in deciding the character of the general patterns of advective drift. In other recent reports of residual drift models of the same or similar areas (Ronday, 1973, Horwood, 1974) this particular aspect of affairs is not as closely examined as we feel it needs to be.

DISCUSSION

We think it is very important to stress at this point that in our view the situation with regard to the relevance or otherwise of the Stokes' drift factor to moored current meter measurements in the North and Irish Seas is of fundamental importance to fisheries oceanographers working there: it is not simply a sacred cow. Is Dooley (1974a) correct, for example, when he suggests "neither drogues nor current meters can adequately predict the movement and dispersal of plankton populations and pollutants"? If he is, are we correct in following Longuet-Higgins, and by implication the Dooley (1974b) paper, in thinking that the Stokes' drift factor as calculated from the phase velocity and amplitude of the tidal wave in a given area is the missing piece of jig-saw? It would seem that we need general agreement among European current-measuring research teams regarding several points. Firstly, we need to assess whether or not Stokes' drift is likely to be of importance in our work. Secondly, if it is of importance we must produce an agreed definition of the Stokes' velocity and of the way in which it can be calculated. Thirdly, we should produce, via the agreed definition, estimates of the Stokes' drift at neap and spring equinoctial tides for grid points covering the whole of the European continental shelf seas at most, and the Irish and North Sea areas at least, so that we can all see the variability of the factor under discussion. Fourthly, we might then feel that we can "zone" the factors that have to be applied after the fashion of the velocity of sound corrections and so reduce our present dilemma to the status of a correction in our computer programs that is akin to, but a little more complicated than, dealing with "magnetic variation" and its effect on current meter compasses.

Finally we need to re-think our field programme. It seems clear to us that we need to approach the problem via "Stokes' triangles" of various dimensions and the use of either shallow water Swallow floats or the labelled tracers suggested by Dooley (1974b).

CONCLUSIONS

1. An ICES Working Group consisting of theoreticians, active modellers and field scientists should be formed to examine the Stokes' drift concept and report within one year as to:-

a. its importance in European Shelf seas;

b. how it should be defined;

c. its magnitude at neap and spring equinoctial tides at points over a grid covering at least the Irish and North Seas and the Channel;

d. the possibility of producing "zones" of common Stokes' drift.

2. Every effort should be made to produce a shelf-seas Swallow float so that reliable field measurements of the Stokes' drift factor can be made in carefully chosen regions.

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ANNEX 1

The formulae used by all the authors mentioned in this paper are derived from a paper by Longuet-Higgins (Longuet-Higgins 1969). With the exception of the new work presented in this paper the starting point is the formula for the Stokes' velocity derived by Longuet-Higgins, ie

$$\underline{U}_s = \overline{\int_{t_0}^{t} \underline{u}\, dt. \quad \underline{u}}, \qquad (1)$$

Where $\underline{U}_s$ is the Stokes' velocity vector, $\underline{u}$ is the Eulerian velocity vector and t is the time; the mean being taken over one or more complete tide cycles. This formula is correct to first order only, but should give a good estimate of the Stokes' velocity provided the excursion of the water particle during one tidal cycle is small compared with the tidal wavelength. This formula was adopted by ourselves (Ramster and Durance, 1973) when using a triangular array of current meter stations to measure the velocity gradient. The Eulerian velocity was taken as the mean of the 3 stations. Only the horizontal motions were considered; the contribution to the Stokes' velocity from the vertical motions was neglected.

If the surface elevation and the horizontal velocity components are assumed to be periodic with zero mean and the vertical motions are neglected, equation (1) can be transformed to:

$$U_s = \frac{\overline{u\zeta}}{h} + \frac{1}{h}\frac{\partial}{\partial y}\left[\overline{hu \int v\, dt}\right]$$

$$V_s = \frac{\overline{v\zeta}}{h} - \frac{1}{h}\frac{\partial}{\partial x}\left[\overline{hu \int v\, dt}\right]$$

where U_s, V_s are the x, y components for $\underline{U}_s$ and u, v are the x, y components of $\underline{u}$. Both Hunter (1972) and Talbot et al (In Prep) use this formulation, neglecting the second term. This is justified in localities where the tidal ellipse is very narrow and the motion approaches one dimensional, but has led to doubts about the general validity of the method.

If in addition to the assumptions of the previous paragraph it is assumed that the motion has the form of a travelling wave, ie the velocity components are functions of (x-ct) and y only, where the x coordinate is in the direction of progagation and c is the phase velocity of the wave, then equation (1) can be reduced to:

$$U_s = \overline{u^2/c} + \overline{\int v\, dt \frac{\partial u}{\partial y}}$$

$$V_s = \int v\,dt\ \overline{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}$$

Again, if the terms involving $\int v\,dt$ are neglected and the wave is assumed sinusoidal, then the formula used by Dooley is obtained:

$$U_s = \tfrac{1}{2} u_m^2/C,$$

$$V_s = 0$$

where u_m is the maximum value of u.

ANNEX 2

The calculated Lagrangian Drift estimates used in figures 2(b) and 3(b) are based on the formula (Longuet-Higgins 1969):

$$\underline{u}(\underline{x}, t) = u(\underline{x}_o, t) + \int_{t_o}^{t} \underline{u}(\underline{x}_o, t)\,dt.\ \underline{\nabla}\,\underline{u}(\underline{x}_o, t), \quad (2)$$

where $\underline{x}$ is the position of a water particle at time t, and $\underline{x}_o$ is its position at the start time t_o. When meaned over a whole number of tidal cycles this formula leads to the formula (1). The Eulerian Drift $\underline{S}_E$ can be defined by

$$\underline{S}_E = \int_{t_o}^{t} \underline{u}(\underline{x}_o, t)\,dt,$$

and the Lagrangian Drift $\underline{S}_L$ by

$$\underline{S}_L = \int_{t_o}^{t} \underline{u}(\underline{x}, t)\,dt.$$

$\underline{S}_E$ and $\underline{S}_L$ are the distances of the particle from the point of release calculated in the Eulerian and Lagrangian sense respectively and can be directly compared with the parachute drogue position.

The integrations were carried out numerically using the hourly means of the velocity from current meter records to obtain hourly values of the Eulerian and Lagrangian Drifts to be compared with the drogue track. The Eulerian velocity at the release point was calculated as a weighted mean of the three current meter records.

A more accurate formula can be obtained by using the Lagrangian Drift $\underline{S}_L$ in the formula (2) instead of the Eulerian Drift $\underline{S}_E$. This formula is referred to in the text as the "Second Order Formula".

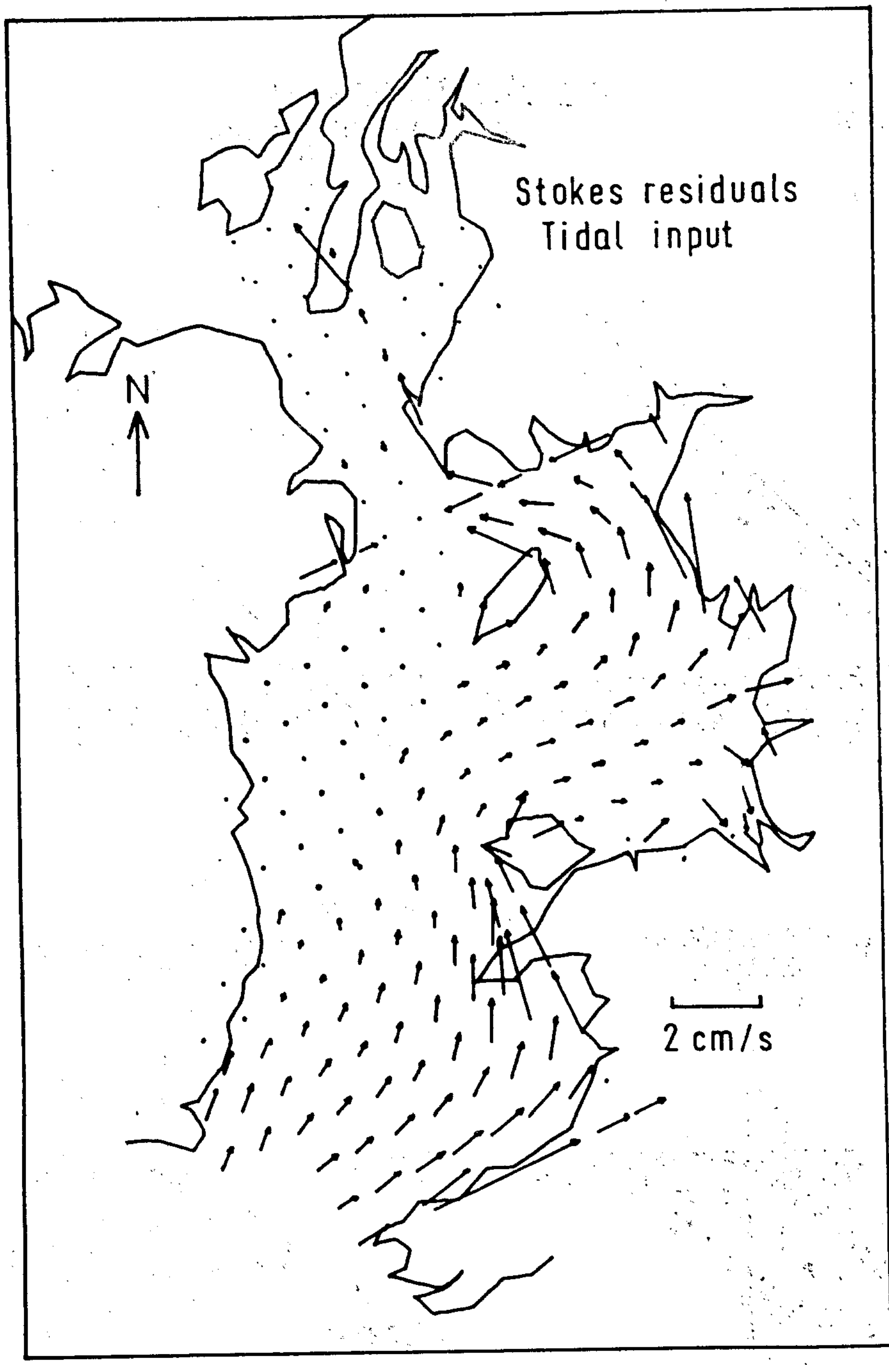

FIGURE 1 Stokes' drift in the Irish Sea calculated from tidal input. (After Hunter 1972)

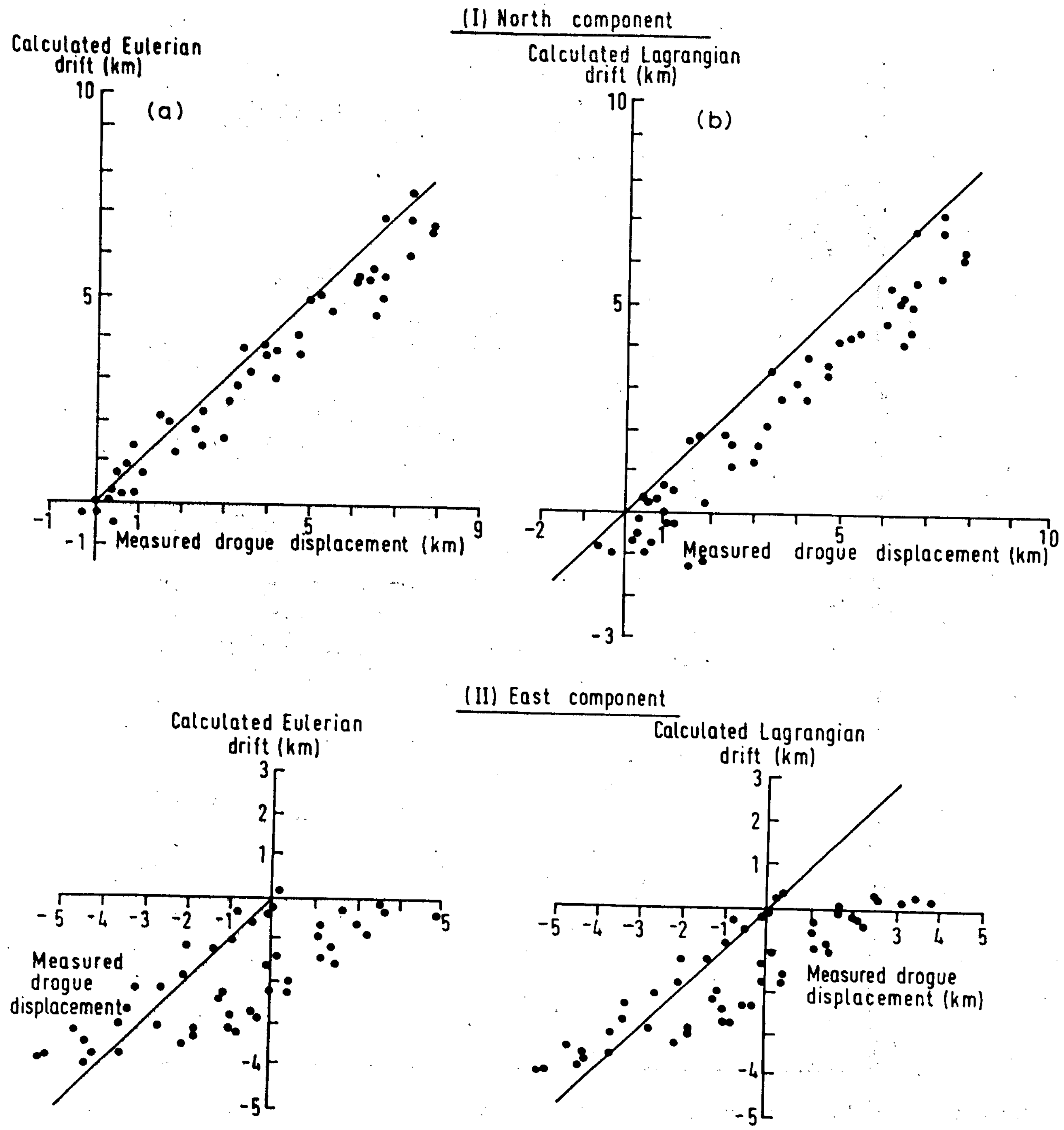

FIGURE 2 A comparison of parachute drogue displacement and (a) Eulerian drift and (b) Lagrangian drift as calculated by the Longuet-Higgins' formula from 45 hours of data collected off Flamborough Head in March 1974.

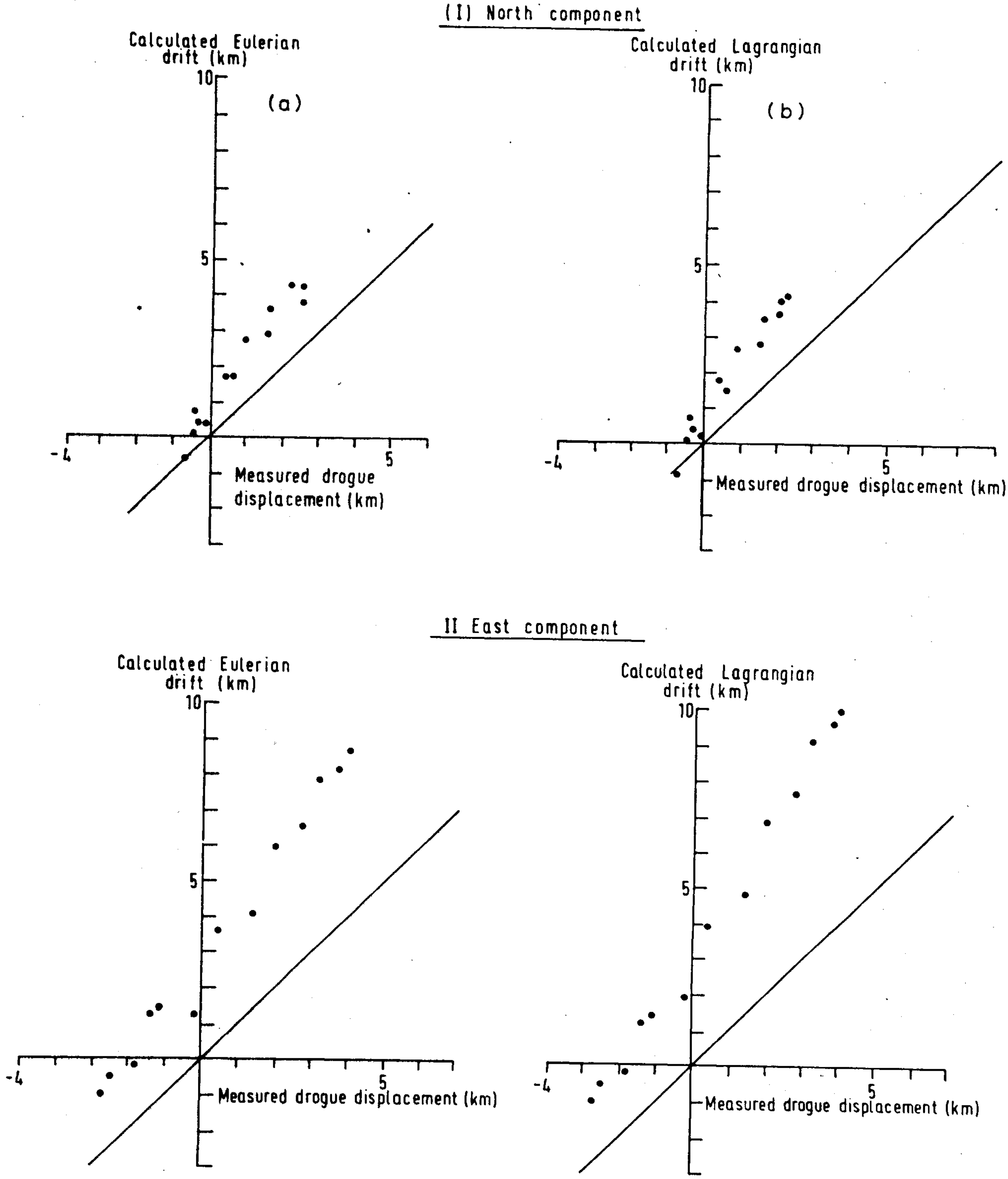

FIGURE 3 A comparison of parachute drogue displacement and (a) Eulerian drift and (b) Lagrangian drift as calculated by the Longuet-Higgins' formula from 13 hours of data collected in the Western Approaches in December 1973.